HOMEWORK FOR CLASS XII (2018-19)

(1). Show that the relation R on the set Z of all integers defined by
$R = \{(a,b): a,b \in Z, a-b, is, divisible, by, 2\}$ is an equivalence relation on Z.

(2). Let f: R$\rightarrow$R be defined by

$$f(x) = \begin{cases} 1 & if \quad x > 0 \\ 0 & if \quad x = 0 \\ -1 & if \quad x < 0 \end{cases}$$ Is f(x) one-one and onto.

(3). If f, g : R$\rightarrow$R are defined respectively by $f(x) = x^2 + 3x + 1, g(x) = 2x - 3$, find
(i) fog (ii) gof (iii) fof (iv) gog

(4). If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$ show that fof(x)=x for all $x \neq \frac{2}{3}$. What is the inverse of f.

(5). Let * be a binary operation on Z defined by
$a * b = a + b - 4, for, all, a, b \in Z$
(i) Show that * is both commutative and associative.
(ii) Find the identity element in Z

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(6). Show that the operation * on Q-{1} , defined by a*b = a + b – ab satisfies the associative and commutative laws, find the identity element , for each element find the inverse.

(7). Let T be the set of all triangles in a plane with R as relation in T given by $R = \{(T_1, T_2) : T_1 \cong T_2\}$. Show that R is an equivalence relation.

(8). If $f(x) = x + 7, and, g(x) = x - 7, x \in R, find, (gof)(7)$.

(9). Is the binary operation *, defined on N, given by $a * b = \frac{a+b}{2}, for, all, a, b \in N$ commutative and associative?.

(10). Let * be a binary operation on Q defined by $a * b = \frac{3ab}{5}$. Show that * is commutative as well as associative. Also find its identity element, if exists.

(11). (i) $\operatorname{cosec}^{-1}(-\sqrt{2})$ (ii) $\tan^{-1}(1) + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$ (iii) $\sec^{-1}(-2)$

(12). Prove that:- $\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{\pi}{2}, x \in \left(0, \frac{\pi}{4}\right)$

(13). Solve for x:- $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$

(14). Prove that:- $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$.

(15). Prove that :- $4\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{70}\right) + \tan^{-1}\left(\frac{1}{99}\right) = \frac{\pi}{4}$

(16). Prove that:- $2\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right] = \cos^{-1}\left(\frac{b+a\cos x}{a+b\cos x}\right)$

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(17)..Find the Principal value of $\sin^{-1}\left(\sin\frac{4\pi}{5}\right) + \cos^{-1}\left(\cos\frac{4\pi}{5}\right)$

(18). Solve: $\tan^{-1}3x + \tan^{-1}2x = \frac{\pi}{4}$

(19). Prove that : $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

(20). Solve for x: $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\pi/4$.

(21). $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}, and, A^2=\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$ then prove that $\alpha+\beta=(a+b)^2$

(22). Find x, If :-

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}=O$$

(21). If A = $\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ show that I + A = $(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

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(22). Use the matrix method to solve the system linear equations. 5

$3x+2y-2z=3$

$x+2y+3z=6$

$2x-y+z=2$

(23). Given that $A=\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}, and, B=\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB. Use this result to solve the following system of linear equations:

$x-y+z=4$

$x-2y-2z=9$

$2x+y+3z=1$

(24). Using properties of determinants find the value of $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$.

(25). If A is a square matrix of order 3 and $|A|=4$ find $|3A|$.

(26). $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}=(a+b+c)^3$

(27) . If $A=\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}.Using A^{-1},solve the following system of equations

$2x-3y+5z=16$

$3x+2y-4z=-4$

$x+y-2z=-3$

(28). If A is a square matrix of order 3 and $|A|=4$ find $|A\,adjA|$.

(29). If A is a square matrix of order 3 such that $|\boldsymbol{adjA}| = \mathbf{81}$ find $|\boldsymbol{A'}|$.

(30). A matrix A of order 3x3 has determinant 4.What is the value of |adjA|.

(31). Using matrix ,solve the following system of linear equations

X+2y-3z=-4, 2x+3y+2z=2, 3x-3y-4z=11.

(32). If A is a square matrix of order 3 and $|A| = 4$ then find the value of $|adj\,A|$ and $|A\,adj\,A|$

(33). If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ find AB. Hence solve the system of equations: $x - y = 3, 2x + 3y + 4z = 17$ *and* $y + 2z = 7$.

1.Discuss the continuity of the function f given by

(33). f(x)=$\begin{cases} (x^3 - 3) & if\ x \le 2 \\ x^2 + 1 & if\ x > 2 \end{cases}$

(34). Find the values of a & b such that the function defined by

F(x)=$\begin{cases} 5 & if\ x \le 2 \\ ax + b & if\ 2 < x < 10 \\ x^2 + 1 & if\ x \ge 10 \end{cases}$ is a continuous function.

(35). Prove that the function f defined by f(x) = $|x - 1|$, x∈R is not differentiable at x=1.

(36). Differentiate $x^{\sin x} + sinx^{\cos x}$ w.r.to x.

(37). If y = $(\tan^{-1} x)^2$

Show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$

(38). If cosy= x cos(a+y) with cosa≠±1 prove that $\frac{dy}{dx} = \frac{cos^2(a+y)}{sina}$

(39). If x= a(cost +t sint) and y= a(sint – tcost), find $\frac{d^2y}{dx^2}$

(40). Differentiate: $\mathbf{cot^{-1}}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$ w. r. to x. where 0< x<π/2.

(41). Find the value of a and b so that following

functions $f(x) = \begin{cases} 1 & if & x \le 3 \\ ax + b & if & 3 < x < 5 \\ 7 & if & x > 5 \end{cases}$ is continuous at x = 3

(42). For what value of K the function is continuous at x=2

$$f(x) = \begin{cases} 2x+1, x < 2 \\ k, \quad x = 2 \\ 3x-1, x > 2 \end{cases}$$

(43). If y= Sin(m $\sin^{-1}$ x) then prove that $(1-x^2)y_2 - xy_1 + m^2y = 0$

(44). Sand is pouring from a pipe at the rate of 12cm^3/sec. the falling sand forms a cone on the ground in such a way that the height of the cone is always one- sixth of the radius of the base. How fast is the height of the sand-cone increasing, when height is 4 cm.

(45). Find the interval in which the function given by $f(x)=x^3-6x^2+6x+15$ is strictly increasing or decreasing.

(46). Find the interval in which the function given by $f(x)=2x^3-15x^2+36x+17$ is strictly increasing or decreasing.

(47). Find the intervals in which the function f given by f(x)=sinx + cosx , 0≤x≤2$\boldsymbol{\pi}$. is strictly increasing or strictly decreasing.

(48). Find the slope of the tangent to the curve $y=x^3-3x+2$ at the point whose x – coordinate is 3.

(49). Find the points at which the tangent to the curve $y= x^3-3x^2-9x+7$ is parallel to the x axis.

(50). Find the equation of the tangents and Normal to the curve

$y=x^4- 6x^3 + 13x^2 - 10x + 5$ at (0,5)

(51). Find the slope of the normal to the curve x=acos$^3\boldsymbol{\theta}$,y=asin$^3\boldsymbol{\theta}$ at $\boldsymbol{\theta}=\frac{\pi}{4}$.

(52). Find the area bounded by the parabola y = $4\boldsymbol{x^2}$,x≥0, the axis of y and lines y= 1 and y=4.

(53). Find area included between the curves $x^2 = 4ay$ and $y^2 = 4ax$, $a > 0$.
(54). Find area of the region : $\{(x, y) \mid x^2 + y^2 \le 1 \le x + y\}$
(55). Using Integration find the area of the triangle ABC whose vertices are A(2,0) ,B(4,5) and C(6,3)
(56). Find the area of the region enclosed between the two circles $x^2 + y^2 = 1$ and $(x-1)^2 + y^2 = 1$.
(57). Sketch the graph of y = $|x+3|$ and evaluate $\int_{-6}^{0} |x+3|\, dx$. What does this integral represent?
(58). Draw a rough sketch of the region enclosed between the circles $x^2 + y^2 = 9$ and $(x - 3)^2 + y^2 = 9$. Using integration find the area of the enclosed region.
(59). Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.
(60). Solve the following differential equation:

$$x\frac{dy}{dx} + y = x\log x, \quad x \ne 0$$

(61). Solve the following differential equation:

$$\cos^2 x\frac{dy}{dx} + y = \tan x,$$

(62). Solve the following differential equation:

$$(x^2 - y^2)\,dx + 2xy\,dy = 0 \quad \text{if } y=1 \text{ when } x=1$$

(63). Solve the following differential equation:

$$\frac{dy}{dx} + y\tan x = \sin x, \quad given that\ \ y = 0 \ \ at \ \ x = \frac{\pi}{3}$$

(64). Solve the following differential equation:

$$(3xy + y^2)\,dx + (x^2 + xy)dy = 0$$

(65). Find the differential equation of the family of all circles touching the x-axis at the origin.
(66). Form the differential equation of family of parabolas having vertex at the origin and axis along positive y-axis.
(67). Find the value of $\boldsymbol{\lambda}$ for which the vector $\vec{a} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ *and* $\vec{b} = \hat{\imath} + \lambda\hat{\jmath} - 3\hat{k}$ ***are perpendicular to each other***.
(68). If p is a unit vector and $(\vec{x} - \vec{p}) \cdot (\vec{x} + \vec{p}) = 80$ ***then find*** $|\vec{x}|$.
(69) If $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ *and* $\vec{b} = 3\hat{\imath} + \hat{\jmath} - 5\hat{k}$, find a unit vector in the direction of $(\vec{a} - \vec{b})$.
(70). Find the angle between two vectors $\vec{a}$ & $\vec{b}$, *if* $|\vec{a}| = 3, |\vec{b}| = 4$ *and* $|\vec{a} \times \vec{b}| = 6$
(71). Find the projection of $\vec{b} + \vec{c}$ ***on*** $\vec{a}$, ***where*** $\vec{a} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ ***and*** $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$.
(72). If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ .Show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$ where $\vec{a} \ne \vec{d}$ & $\vec{b} \ne \vec{c}$
(73). Using vector find the area of the triangle with vertices A(2, 3, 5), B(3, 5, 8) And C(2, 7, 8).
(74). Find the value of $\boldsymbol{\lambda}$ which makes the vectors coplanar, where $\vec{a} = -4\hat{\imath} - 6\hat{\jmath} - 2\hat{k}$, $\vec{b} = -\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$ ***and*** $\vec{c} = -8\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$.

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(75). If $\vec{a}, \vec{b}$ ***and*** $\vec{c}$ are unit vectors such that

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.

(76). Show that the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$, $3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$ form the vertices of a right angled triangle.
(77). Find a unit vector perpendicular to each of the vector $(\vec{a} + \vec{b})$ ***and*** $(\vec{a} - \vec{b})$ where $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

(78). Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

(79). Find the distance of the point $(2, -1, -4)$ from the plane $3x - 4y + 12z = 9$.

(80). Find 'α' when vectors i - j + k & i+ α j + 2k are

(i) Perpendicular (ii) Parallel

(81).(3). Find the equation of line passing through the point (-2, 4, -5) and parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+6}{8}$

(82). Find the equation of the plane which contains line of intersection of planes $\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$, $\vec{r}.(2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which passes through the Point (1,0,-2).

(83). Find the value of 'P' so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other.

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(84). Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ *on the vector* $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

(85). Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-4}{-2} = \frac{z-7}{1}$

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(86). Find the equation of the plane passing through the line of intersection of the planes $x - 2y + z = 1$ ***and*** $2x + y + z = 8$ and parallel to the line with direction ratios 1, 2, 1.Also find the perpendicular distance of the point $P(3, 1, 2)$ from the plane.

(87). Find the image of $(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

(88). Show that the lines $\vec{r} = (5\hat{i} + 7\hat{j} - 3\hat{k}) + \lambda(4\hat{i} + 4\hat{j} - 5\hat{k})$ and $\vec{r} = (8\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(7\hat{i} + \hat{j} + 3\hat{k})$ Intersect. Find the point of intersection.

(89) Find the equation of the plane passing through the point (−1, 3, 2) and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

(90). Find the vector equation of the line passing through the point (1, 2, − 4) and perpendicular to the two lines:

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

(90). Find the equation of the plane that passes through three points.

(1, 1, −1), (6, 4, −5), (−4, −2, 3)

(90). Find the angle between the planes whose vector equations are

$\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$.

(91).A local television network is faced with the following problem. It has been found that programme A with 12 minutes of music and 1 minute of advertisement draws 15000 viewers while programme B with 15 minutes of music and 1 minute of advertisements draws 17000 viewers. Within one week the advertiser visits that at least 7 minutes be devoted to his advertisement and TV network can afford not more than 90 minutes of music. How many times/weeks should each programme be given in order to obtain the maximum number of viewers?

(a). Formulate the problem as a linear programming

(b). Solve it by graphic method

(92). A dealer in rural area wishes to purchase a number of sewing machines. He has only `5,760.00 to invest and has space for at most 20 items. An electronic sewing machine costs him `360.00 and a manually operated sewing machine `240.00. He can sell an Electronic Sewing Machine at a profit of `22.00 and a manually operated sewing machine at a profit of `18.00. Assuming that he can sell all the items that he can buy how should he invest his money in order to maximize his profit. Make it as a linear programming problem and solve it graphically.

(93).A toy company manufactures two types of dolls A and B. Market tests shows that the combined production level should not exceed 1200 dolls per week and the demand of dolls B is at most half of dolls A. Further the production level of type A can exceed 3 times the production of type B by at most 600 units .If the company makes profit of Rs.12 and Rs.16 per doll respectively on dolls A and B. How many of each should be produced weekly in order to maximize the profit?.

(94).A factory owner purchased two types of machines, electronic and manually operated requirements and the limitations for the machines are as follows:

	Area occupied by the machine	Labor force on each machine	Daily output
Electronic	1000 Sq.m	12 men	60
Manually	1200 Sq.m	8 men	40

He has maximum area of 8400 m^2 available, and 88 skilled labors who can operate both the machines of each type should he buy to maximize the daily output? Keeping unemployment in mind justify the values to be promoted for selection of the manually operated machine.

(95). An aero plane can carry a maximum of 200 passengers. A profit of Rs.400 is made on each first class ticket and a profit of Rs. 300 is made on each second class ticket. The airline reserves at least 20 seats for first class. However at least four times as many passengers prefer to travel by second class than by first class. Determine how many tickets of each type must be sold to maximize profit for the airline. Form an L.P.P. and solve it graphically.

(96). Given two independent events A & B such that P(A)= 0.3 & P(B)=0.6 find
(i) P(A &B) P(A or B) (iii) P(A and not B) (iv) P(neither A nor B)

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(97). If a family has two children. What is the conditional probability that both are girls, if given that
(i) The youngest is a girl.
(ii) At least one is girl.

(98). There are 40 workers in a factory who are participating in strike for increase in wages. Out of them 5 workers are sincere to management and believe that management know each thing and do the needful. Management selects 2 workers at random out of them. Write the probability distribution for the selected worker who is sincere. Also find the mean of the distribution. Write the important of sincerity in service.

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(99). In bag A there are 5 red balls and 3 white balls and in bag B there are 3 red balls and 5 white balls. If a ball is drawn from one of these two bags and found to be red, find the probability that it is drawn from bag A.

(100). A and B toss a coin alternately till one of them gets a head and wins the game. If A starts the game, find their respective probabilities of winning.

(101). If a fair coin is tossed 10 times, find the probability of
(a)Exactly six heads.(b) At least six heads. (c) At most six heads.

(102). Evaluate: $\int \frac{1}{\cos(x-a).\cos(x-b)} dx$

(103). Evaluate: $\int \frac{1}{x^2(x^4+1)^{\frac{3}{4}}} dx$

(104). Evaluate: $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

(105). Evaluate: $\int \frac{1}{x(1+logx)\,(2+logx)} dx$

(106). Evaluate: $\int \frac{5x+3}{\sqrt{x^2+4x+10}}\ dx$

(107). Evaluate: $\int_2^3 x^2\ dx$, using integral as a limit of sum.

(108). Evaluate: $\int \boldsymbol{e^x(sinx + cosx)\ dx}$

(109). Evaluate: $\int \boldsymbol{e^x\left(logx + \frac{1}{x}\right)\ dx}$

(110). Evaluate: $\int \boldsymbol{\frac{1}{x(1+logx)\,(2+logx)}\ dx}$

(111). Evaluate: $\int \boldsymbol{\frac{2x+5}{\sqrt{x^2+3x+2}}\ dx}$

(112). Evaluate: $\int_{-5}^{5} \boldsymbol{|x+2|\ dx}$

(113). Evaluate: $\int_2^3 (\boldsymbol{x^2} + x)\ \boldsymbol{dx}$, using integral as a limit of sum.

(114). Evaluate: $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}\ \boldsymbol{dx}$, using the property of definite integral.

(115). Evaluate: $\int_0^{\frac{\pi}{4}} \boldsymbol{\log(1 + tanx)\ dx}$, using the property of definite integral.

(116). Evaluate: $\int_0^{\pi} \boldsymbol{\frac{x}{a^2cos^2x+b^2sin^2x}\ dx}$, using the property of definite integral.

(117).Evaluate: $\int (2x+3)\sqrt{4x^2+5x+6}\ dx$.

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(118). Evaluate $\int \frac{\sec^2 x}{3+\tan x} dx$

(119). Evaluate $\int \frac{3x+2}{(x-1)(2x+3)} dx$

(120). Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$