HOLIDAY HOMEWORK (AUTUMN BREAK)
CLASS:- XII (2017-18)

COUNTINUITY AND DIFFERENTIABILITY

(1). Find $\frac{dy}{dx}$ if y= $(\log x)^x + x^{\log x}$

(2). If y= Sin(m $\sin^{-1}$ x) then prove that $(1-x^2)y_2 - xy_1 + m^2y = 0$

(3). Find $\frac{dy}{dx}$ if y= $\tan^{-1}\left(\frac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}}\right)$

(). If $y = (x)^{\sin x} + (\sin x)^{\cos x}$, find $\frac{dy}{dx}$

(5). If $y = \left(\tan^{-1} x\right)^2$, show that $\left(x^2+1\right)^2 y_2 + 2x\left(x^2+1\right)y_1 = 2$

(6). If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$

(7).If $f(x) = \begin{cases} 3ax+b, & if \quad x>1 \\ 11, & if \quad x=1 \\ 5ax-2b, & if \quad x<1 \end{cases}$ is continuous at $x = 1$, find the value of a and b.

(8). Differentiate $\tan^{-1}\frac{2x}{1-x^2}$ with respect to $\sin^{-1}\frac{2x}{1+x^2}$

APPLICATION OF DERIVATIVE

(9). Sand is pouring from a pipe at the rate of 12cm^3/sec. the falling sand forms a cone on the ground in such a way that the height of the cone is always one- sixth of the radius of the base. How fast is the height of the sand-cone increasing, when height is 4 cm.

(10). Find the interval in which the function given by $f(x)=2x^3-15x^2+36x+17$ is strictly increasing or decreasing.

(11).Prove that the function f given by $f(x) = x^2-x+1$ is neither strictly increasing nor strictly decreasing on(-1 ,1).

(12). Find the intervals in which the function f given by
f(x)=sinx + cosx , $0 \le x \le 2\pi$. is strictly increasing or strictly decreasing.

(13). Find the slope of the tangent to the curve $y=x^3-3x+2$ at the point whose x – coordinate is 3.

(14). Find the points at which the tangent to the curve $y= x^3-3x^2-9x+7$ is parallel to the x axis.

(15). Find the equation of the tangents and Normal to the curve

$y=x^4- 6x^3 + 13x^2 - 10x + 5$ at (0,5)

(16).Find the slope of the normal to the curve $x=a\cos^3\theta$,$y=a\sin^3\theta$ at $\theta=\frac{\pi}{4}$.

INTEGRALS

(17). Evaluate: $\int_{-5}^{5} |x+2| \, dx$

(18). Evaluate: $\int_2^3 x^2 \, dx$, using integral as a limit of sum.

(19). Evaluate: $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}} \, dx$, using the property of definite integral.

(22). Evaluate: $\int_0^{\frac{\pi}{4}} \log(1+\tan x) \, dx$, using the property of definite integral.

(20). Evaluate: $\int_0^{\pi} \frac{x}{a^2\cos^2 x+b^2\sin^2 x} \, dx$, using the property of definite integral.

(21). Evaluate: $\int_0^{\frac{\pi}{2}} \log\sin x \, dx$, using the property of definite integral.

APPLICATION OF INTEGRALS

(22). Draw a rough sketch of the region enclosed between the circles $x^2 + y^2 = 9$ and $(x-3)^2 + y^2 = 9$. Using integration find the area of the enclosed region.

(23). Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

(24).Using integration find the area of region bounded by the triangle whose vertices are

(– 1, 0), (1, 3) and (3, 2).

(25). Find the area lying above x-axis and included between the circles $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

(26). Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(27). Find area included between the curves $x^2 = 4ay$ and $y^2 = 4ax$, $a > 0$.

DIFFERENTIAL EQUATIONS

(28). Solve $e^{\frac{dy}{dx}} = x^2$

(29). Determine the order and degree of the following differential equations

$$Y = x.\frac{dy}{dx} + \sqrt{a^2}\left(\frac{dy^2}{dx}\right) + b^2$$

(30). State whether y= $e^{-x}(x+a)$ is the solution of the differential equation

$$\frac{dy}{dx} + y = e^{-x}$$

(31). Solve the following differential equation

$\frac{dy}{dx} = y \sin 2x$ given that y(0)=1

(32). Find the differential equation of the family of curves given by

$x^2 + y^2 = 2ax$

(33). Solve the following differential equation:

$$(3xy + y^2)\,dx + (x^2 + xy)dy = 0$$