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HOLIDAY HOMEWORK (SUMMER VACATION)
CLASS:- XII (2018-19)

(RELATIONS AND FUNCTIONS)

(1). If f(x) is an invertible function, find the inverse of $f(x) = \frac{3x-2}{5}$.

(2). Let T be the set of all triangles in a plane with R as relation in T given by $R = \{(T_1, T_2) : T_1 \cong T_2\}$. Show that R is an equivalence relation.

(3). If $f(x) = x+7, and, g(x) = x-7, x \in R, find, (gof)(7)$.

(4). Is the binary operation *, defined on N, given by $a*b = \frac{a+b}{2}, for, all, a, b \in N$ commutative and associative?.

(5). Show that the relation R defined by $R = \{(a,b) : a-b, is, divisible, by, 3; a, b \in N\}$ is an equivalence relation.

(6). Let $f : N \to N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, if, n, is, odd \\ \frac{n}{2}, if, n, is, even \end{cases}$, Find whether the function f is bijective.

(7).). Let f: R$\to$R be defined by

$$f(x) = \begin{cases} 1 & if \quad x > 0 \\ 0 & if \quad x = 0 \\ -1 & if \quad x < 0 \end{cases}$$ Is f(x) one-one and onto.

(8). If the binary operation * defined on Q is defined as $a*b = 2a + b - ab$ for all $a, b \in Q$, find the value of 3*4.

(9) Show that the relation R defined by $(a,b)R(c,d) = a+d = b+c$ on the set NxN is an equivalence relation.

(10). Show that the relation R in the set of real numbers defined as $\{R = (a,b) : a \le b^2\}$ is neither reflexive, nor symmetric, nor transitive.

(11). If f, g : R$\to$R are defined respectively by $f(x) = x^2 + 3x + 1, g(x) = 2x - 3$, find

(i) fog (ii) gof (iii) fof (iv) gog

(12). Let * be a binary operation on Q defined by $a*b = \frac{3ab}{5}$. Show that * is commutative as well as associative. Also find its identity element, if exists.

(13). Show that the function $f : R \to R$ defined as $f(x) = 2x + 1$ is one-one and onto.

(14). Show that the function $f : R \to R$ defined as $f(x) = x^2$ is neither one-one nor onto.

(15). If $f : R \to R$ be defined by $f(x) = (3 - x^3)^{1/3}$ then find $(fof)(x)$.

(INVERSE TRIGONOMETRIC FUNCTIONS)

(16). Write the range and domain of each trigonometric functions

(17). Find the principal value of the following inverse trigonometric functions

(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (ii) $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ (iii) $\sin^{-1}(-1)$ (iv) $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

(v) $\operatorname{cosec}^{-1}(-\sqrt{2})$ (vi) $\tan^{-1}(1) + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$ (vii) $\sec^{-1}(-2)$

(18). Prove that:- $\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{\pi}{2}, x \in \left(0, \frac{\pi}{4}\right)$

(19). Solve for x:- $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$

(20). Prove that:- $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$.

(21). Prove that :- $4\tan^{-1}\left(\frac{1}{5}\right)-\tan^{-1}\left(\frac{1}{70}\right)+\tan^{-1}\left(\frac{1}{99}\right)=\frac{\pi}{4}$

(22). If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, prove that $x+y+z=xyz$

(23). If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$, prove that $xy+yz+zx=1$

(24). Prove that:- $2\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right]=\cos^{-1}\left(\frac{b+a\cos x}{a+b\cos x}\right)$

(25). Solve:- $\sin\left[2\cos^{-1}\left\{\cot\left(2\tan^{-1}x\right)\right\}\right]=0$

(MATRICES)

(26). Construct a 3x3 matrix whose elements are given by $a_{ij}=\frac{2i+3j}{2}$.

(27). Construct a matrix of order 2x2 whose general element is given by $\left[\frac{i}{j}\right]$ where [x] denotes the greatest integer function not exceeding x.

(28). Solve the matrix equation $2\begin{bmatrix} x & y \\ z & t \end{bmatrix}+3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}=3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

(29). Let $A=\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix},, B=\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, and, C=\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$. Find each of the following:

(i) $A+B$ (ii) $A-B$ (iii) $3A-C$ (iv) AB (v) BA

(30). If $A=\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}, and, I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ find K so that $A^2=KA-2I$.

(31). If $A=\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3-6A^2+7A+2I=O$.

(32). If $\begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}, and, B=\begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$ then verify that

(i) $(A+B)'=A'+B'$ (ii) $(A-B)'=A'-B'$ (iii) $(AB)'=B'A'$

(33). Find the values of a and b such that matrix $A=\frac{1}{3}\begin{bmatrix} a & -2 & -2 \\ -2 & -1 & b \\ -2 & b & -1 \end{bmatrix}$ satisfies $AA'=I$

(34). Express the matrix $A=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

(35). By using elementary operations, find the inverse, if exists of the following:

(i) $A=\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ (ii) $A=\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 3 \\ 1 & -2 & -1 \end{bmatrix}$ (iii) $A=\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$